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CAS we can! – But should we?

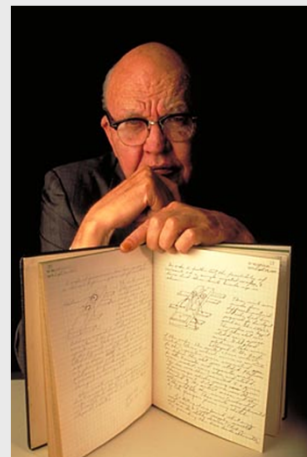
The Integration of Symbolic Calculators into Mathematics Lessons

1. New Technologies – Expectations
 2. 2011
 3. The – longstanding – M³-Project
 4. A competence model for working with SC (CAS)
 5. A view into the future
-

New Technologies – Expectations



Texas Instrument
Weight : 1,5 kg



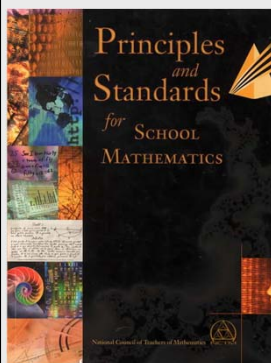
Jack Kilby



Expectations of many math educators 1972ff

Calculators ...

- ... allow experimental activities in the frame of discovery learning and problem solving
- ... give a numerical basis for concept formation
- ... allow the integration of authentic real-life problems into the classroom
- ... release students from algorithmic calculations ...
- will bring far-reaching changes of the goals of mathematics in the classroom.



NCTM-Standards (1989, 2000)

The Technology Principle:

“Calculators and computers are reshaping the mathematical landscape ... Students can learn more mathematics more deeply with the appropriate and responsible use of technology.”



TI-92 (1995),



Casio FX 2.0 (1999)

“The power and flexibility of technology can help **change the focus of school mathematics** completely The student will be no longer a calculator ... he/she will be an **organizer of strategies** and an **interpreter of results** ...”

Many math educators 1990ff



2011



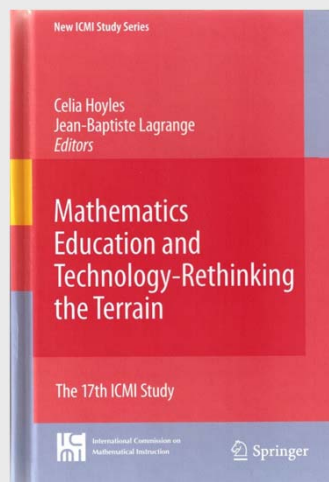
CAS



Greater emphasis on...

- ... graphical representations
- ... working (and thinking) in modules
- ... working individually and cooperatively
- ... a variety of problem solving strategies
- ...

- **But:** The use of SC (CAS) is – world wide – not overwhelming!



ICMI Study 17:



CAS

.... powerful tool!



- **But:** We underestimated students' and teachers' technical difficulties to handle the device.
-



- **And:** It is difficult to prove the benefit of SC in real classroom situations. We need more long-standing empirical investigations!

Important Research Questions...

- Does the use of SC lead to a **better understanding?**
 - Does the use of SC **change students' and teachers' beliefs** concerning the essence of mathematics in school?
-

The – longstanding – M³-Project

States in Germany



Bavaria



2005/06 2006/07 2008/09

10th - 11th - 12th grade

300 ... 1000 students



*) Model-Project-Media-Mathematics



Changes in core mathematical paper and pencil abilities:
*solving equations, manipulations of terms, interpreting
graphs?*

SC-classes Control classes

Pre-test Post-test

No differences



Research question: What is the benefit of the SC use?



Test with SC



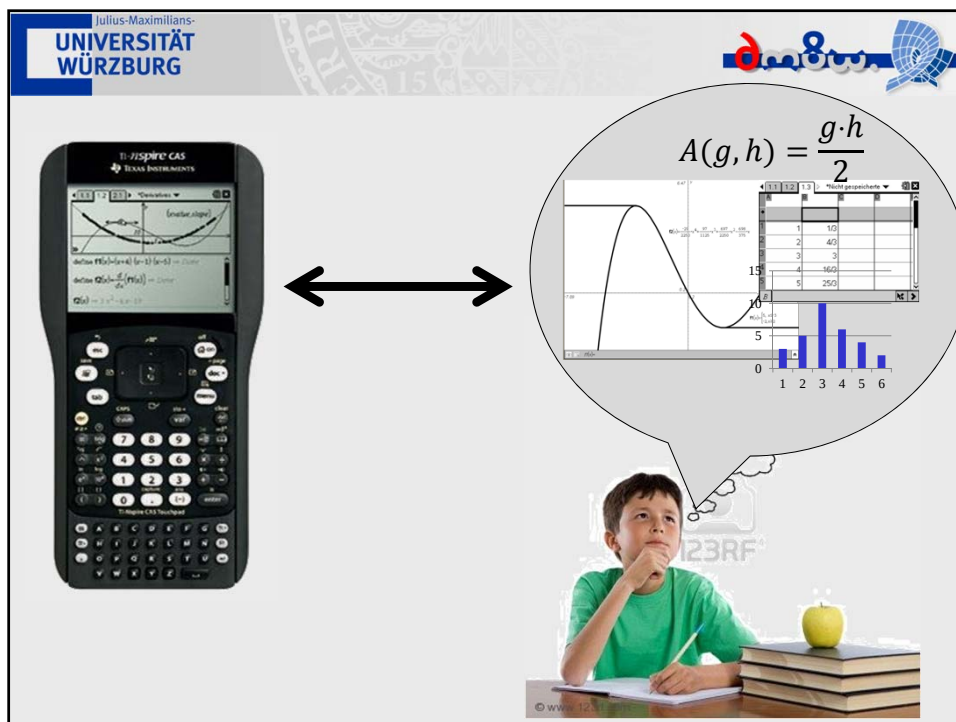
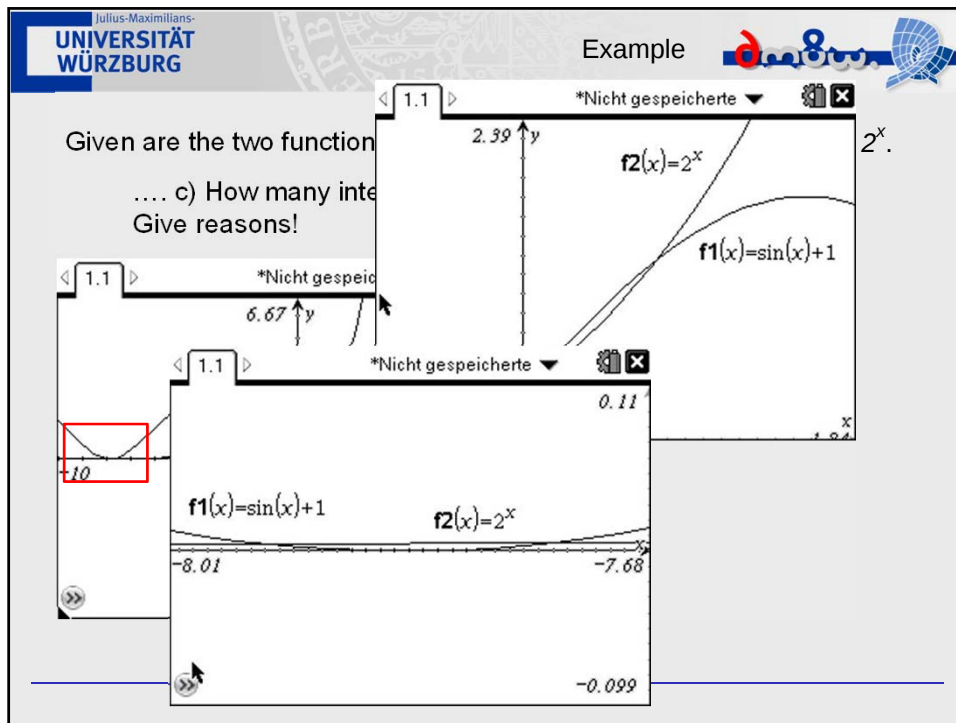
Given are the two functions f and g with $f(x) = \sin(x) + 1$ and $g(x) = 2^x$.

.... c) How many intersection points do the graphs have?
Give reasons!

$\text{solve}(\sin(x)+1=2^x, x)$

$x=-70.6858$ or $x=-70.6858$ or $x=-2.23478$ or $x=0$ or $x=.749645$

⚠ Some more solutions may exist



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Example



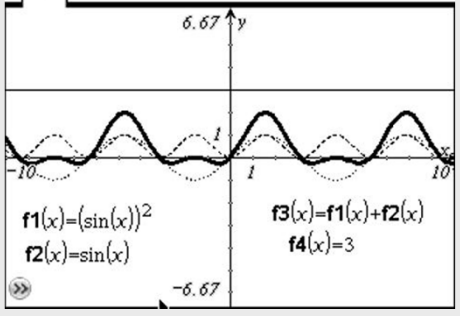
Are there any x-values, that fulfill the equation $\sin^2(x) + \sin(x) = 3$, $x \in \mathbb{R}$?
Give reasons!

1.1 1.2 > *Aufgabe5

solve((sin(x))^2+sin(x)=3,x) false

1/99

1.1 > *Nicht gespeicherte



$f1(x) = (\sin(x))^2$
 $f2(x) = \sin(x)$
 $f3(x) = f1(x) + f2(x)$
 $f4(x) = 3$

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A competence model for working with

Functions

and

SC (CAS)



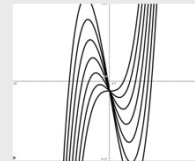
1. Level: Intuitive Understanding

2. Level: Conceptual Understanding



3. Level: Relational Understanding

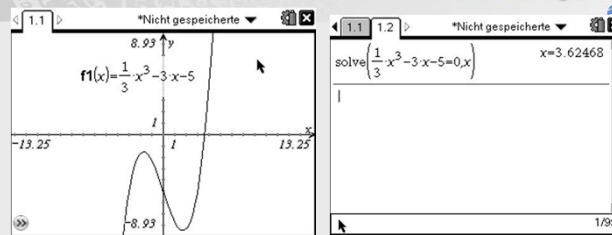
$$f(x) = \frac{1}{3}x^3 + a \cdot x - 1$$



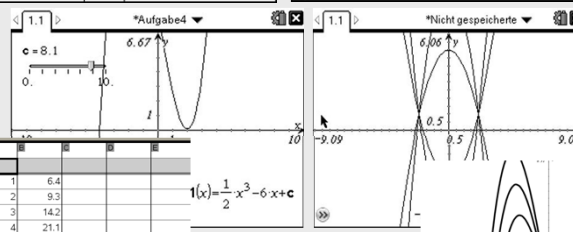
4. Level: Structural Understanding

$$\begin{aligned} f(x) &= \sin(x) \\ g(x) &= ax^2 \\ h(x) &= f(x) + g(x) \end{aligned}$$

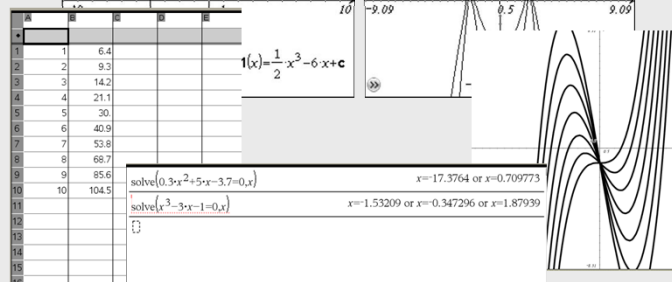
Static Mode



Dynamic Mode



Multi-R-Mode



		UF-TC – Competence Model			
Multi Mode					
Dyna Mode					
Static Mode					
TC	UF	Intuitive U.	Conceptual U.	Relational U.	Structural U.



Problem



Given is the function f_c with $f_c(x) = \frac{1}{2}x^3 - 6x + c$, $c \in \mathbb{R}$. For which values of c does f_c have exactly one zero? Give reasons!

1.1

*Nicht gespeicherte

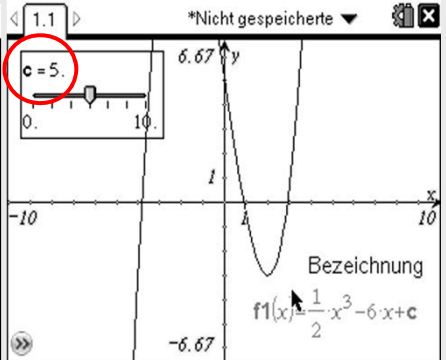
$$\text{solve}\left(\frac{1}{2}x^3 - 6x + c = 0, x\right)$$

$$x = 4 \cdot \cos\left(\frac{\sin^{-1}\left(\frac{c}{8}\right)}{3} + \frac{\pi}{6}\right) \text{ or } x = -4 \cdot \sin\left(\frac{\sin^{-1}\left(\frac{c}{8}\right)}{3} + \frac{\pi}{6}\right)$$

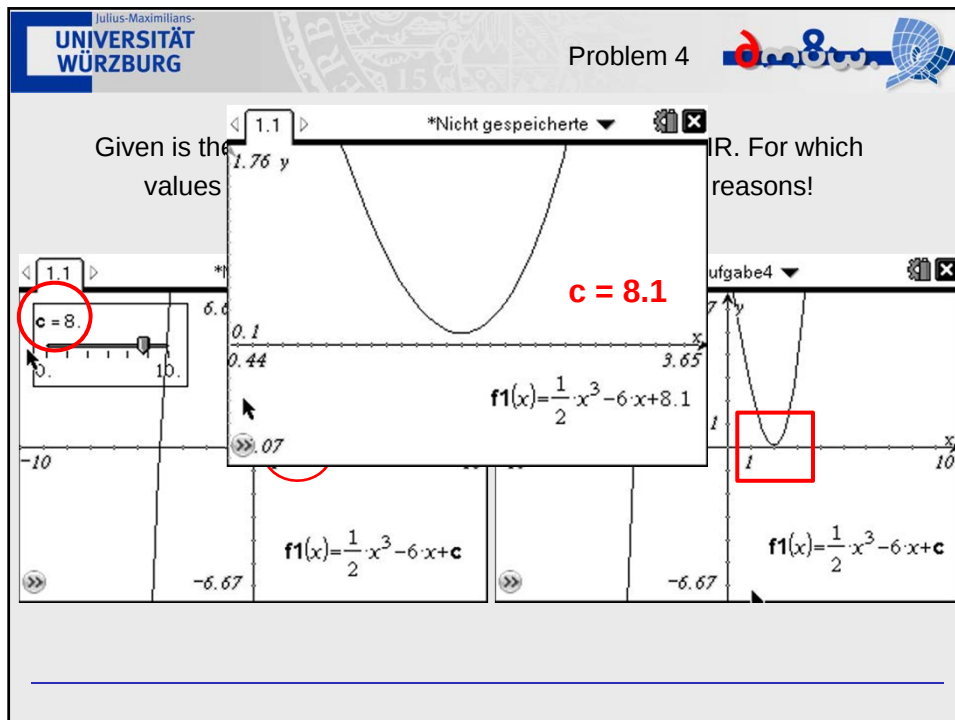
Die Eingabe enthält undefinierte Parameter. Das ...

1.1

*Nicht gespeicherte



[Video](#)



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Problem 4

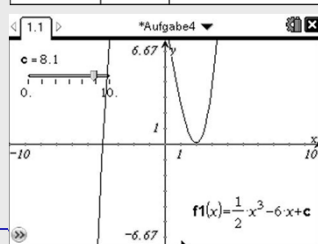
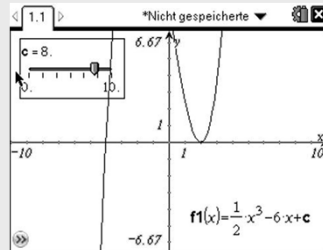
Correct answer: For $x < -8$ or $x > 8$

Wrong answers:

- For all values **from 8** the function has one zero.
- Slider $c = 8.1$. For $c = 8.1$ (and more) the function $f_c(x)$ has only one zero.
- $c > 8.1$ the function has one zero
- $c \geq 8.1$



Given is the function f_c with $f_c(x) = \frac{1}{2}x^3 - 6x + c$, $c \in \mathbb{R}$. For which values of c does f_c have exactly one zero? Give reasons!



```

1.1 *Nicht gespeicherte
Define f(x)=1/2*x^3-6*x+8 Fertig
solve(f(x)=0,x) x=-4 or x=2
Define g(x)=1/2*x^3-6*x+8.1 Fertig
solve(g(x)=0,x) x=-4.00555
|
4/99

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A – short – view into the future



The World in 100 years

Arthur Brehmer (Ed.), The World in 100
Years, Berlin 1910,

The wireless century (Robert Sloss): ...
The telephone in the vest pocket (p.
35ff)



1. The meaning of hand held technology increases.
2. SC (CAS) cannot be seen as isolated tools or media. They have to be integrated into the variety of digital and traditional tools or media.
3. **Connectivity** will be the key word in the future

Algebra ↔ Geometrie

2D-geometry ↔ 3D-geometry

Calculus ↔ Linear Algebra ↔ Stochastics

Mathematics ↔ Physics ↔ ...

